Measuring the Top Quark Mass

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UC Berkeley Qualifying Exam

November 21, 2003

- Why measure the top mass?
- What we've done so far – Run I style CDF measurement
- What I'm working on now – D0-style matrix element method
- Prospects for D0-style method at CDF, including work on transfer functions

- SM so far very successful
  - Predicted W, Z masses
  - Compatible with a huge array of experimental data
- SM consistency checked to high precision
- A few loose ends tied up in last ten years
  - Top quark, ν_τ
- Ongoing exploration
  - Nature of ν's
  - CKM matrix and CP violation
  - H boson still not observed
- Tevatron unlikely to discover H with only 4 fb^{-1}
- Top a fundamental particle
  - Yukawa coupling a fundamental parameter of SM

FERMIONS — matter constituents — spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3

BOSONS — force carriers — spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W$^-$	80.4	−1
W$^+$	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

EW (EWSB) spin = 0

| H | | ? | | 0 |

Radiative Corrections

Fermions affect couplings

$$G_{\text{F}} = \frac{\pi\alpha}{\sqrt{2}m_{\text{W}}^2 \sin^2\theta_{\text{W}}} \frac{1}{(1 - \Delta r)}$$

$$\Delta r = \Delta\alpha + \Delta r_{\text{w}}$$
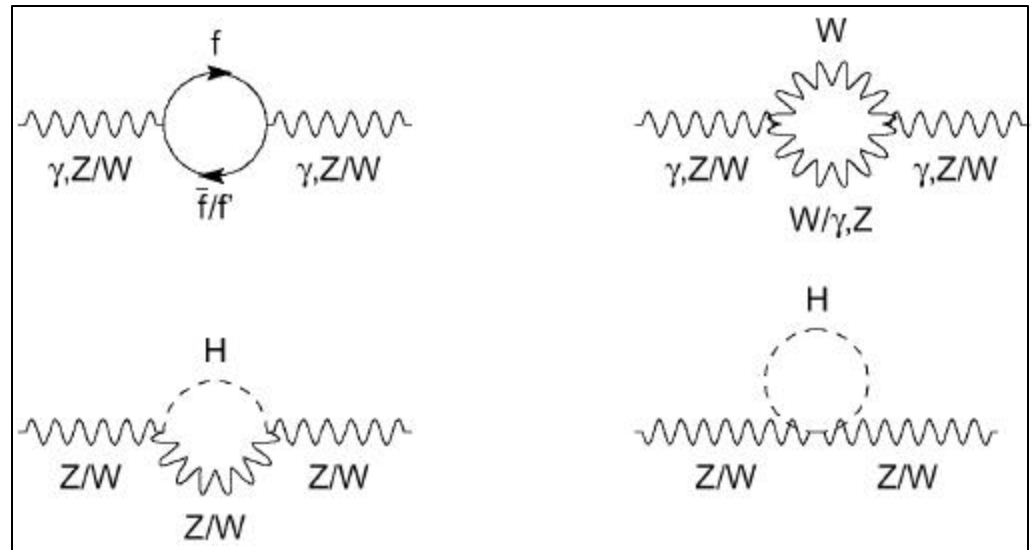
$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)} = \frac{\alpha(0)}{1 - \Delta\alpha_{lept}(s) - \Delta\alpha_{top}(s) - \Delta\alpha_{had}^{(5)}(s)}$$

$$\Delta r^{\text{t}} = -\frac{3G_{\text{F}}m_{\text{W}}^2}{8\sqrt{2}\pi^2} \frac{m_{\text{t}}^2}{m_{\text{W}}^2} \frac{\cos^2\theta_{\text{W}}}{\sin^2\theta_{\text{W}}} + \cdots$$

$$\Delta r^{\text{H}} = \frac{11}{3} \frac{G_{\text{F}}m_{\text{W}}^2}{8\sqrt{2}\pi^2} \left(\ln \frac{m_{\text{H}}^2}{m_{\text{W}}^2} - \frac{5}{6} \right) + \cdots$$

P. Renton hep-ph/0206231

Precision Electroweak

- **High-precision measurements of EW observables**
  - LEP I, SLD
  - LEP II, SLD w/ polarized beams
  - Tevatron
  - νN scattering (NuTeV)
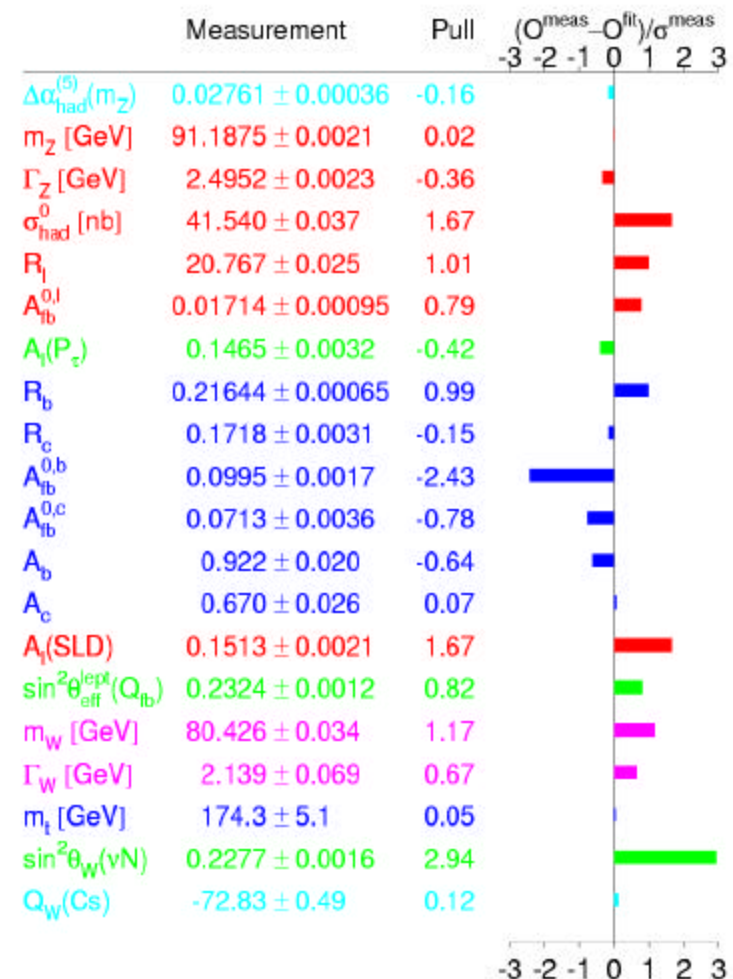 - Atomic Physics

- **Can "predict" top mass**
  - '94 data at least consistent with first m_t measurement
  - Today LEP plus LEP II 180^{+13}_{-11} GeV
  - Today all Z pole 171^{+11}_{-9} GeV
  - Today global fit $174.0^{+4.5}_{-4.4}$ GeV

- **Can predict H mass**
  - 96^{+60}_{-38} GeV, <219 GeV 95% CL

Bob Clare WIN '03

Winter 2003

	Measurement	Pull	$(O^{meas}-O^{fit})/\sigma^{meas}$ -3 -2 -1 0 1 2 3
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02761 ± 0.00036	-0.16	
m_Z [GeV]	91.1875 ± 0.0021	0.02	
Γ_Z [GeV]	2.4952 ± 0.0023	-0.36	
σ^0_{had} [nb]	41.540 ± 0.037	1.67	
R_l	20.767 ± 0.025	1.01	
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.79	
$A_l(P_\tau)$	0.1465 ± 0.0032	-0.42	
R_b	0.21644 ± 0.00065	0.99	
R_c	0.1718 ± 0.0031	-0.15	
$A^{0,b}_{fb}$	0.0995 ± 0.0017	-2.43	
$A^{0,c}_{fb}$	0.0713 ± 0.0036	-0.78	
A_b	0.922 ± 0.020	-0.64	
A_c	0.670 ± 0.026	0.07	
A_l(SLD)	0.1513 ± 0.0021	1.67	
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.82	
m_W [GeV]	80.426 ± 0.034	1.17	
Γ_W [GeV]	2.139 ± 0.069	0.67	
m_t [GeV]	174.3 ± 5.1	0.05	
$\sin^2\theta_W(\nu N)$	0.2277 ± 0.0016	2.94	
$Q_W(Cs)$	-72.83 ± 0.49	0.12	

-3 -2 -1 0 1 2 3

LEPEWWG/2003-01

LEPEWWG/2003-01

m_W Also Key
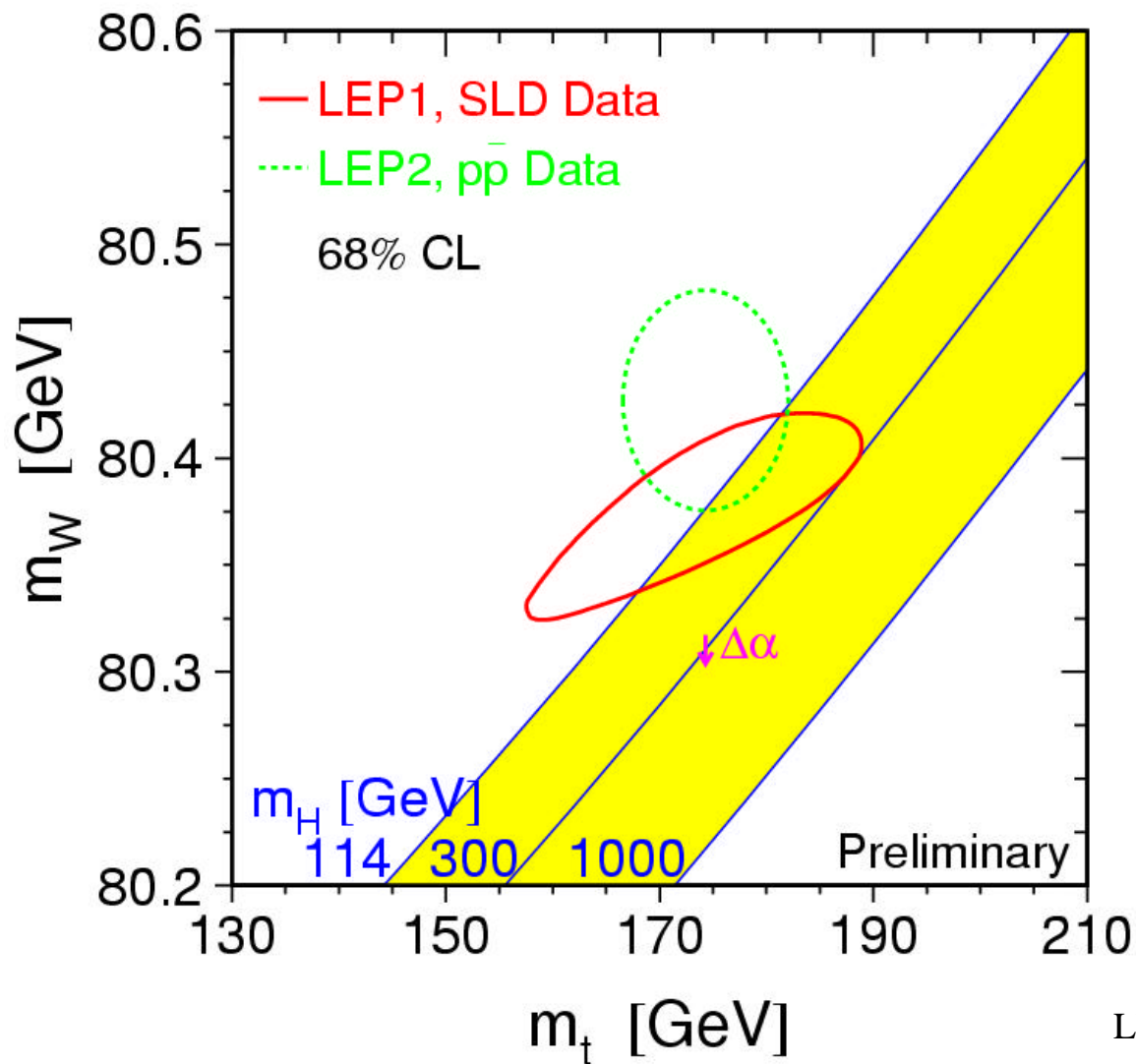
Top contribution

Preliminary CDF Run II Results

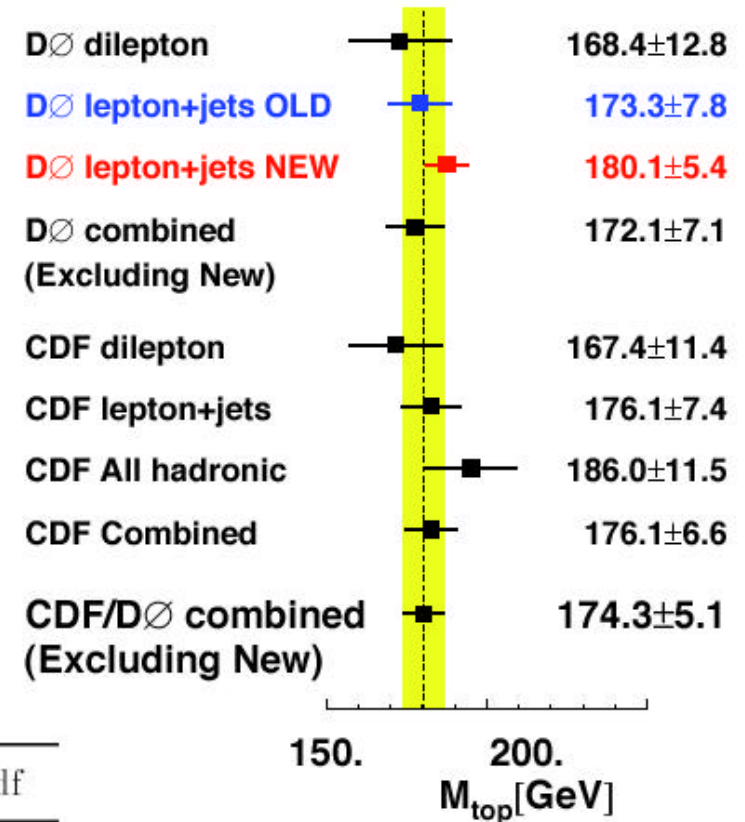$$m_t = 177.5^{+12.7}_{-9.4} \pm 7.1 (\ell + \text{jets})$$
$$m_t = 175.0^{+17.4}_{-16.9} \pm 7.9 (\ell\ell)$$

New (Preliminary) D0 Run I Result
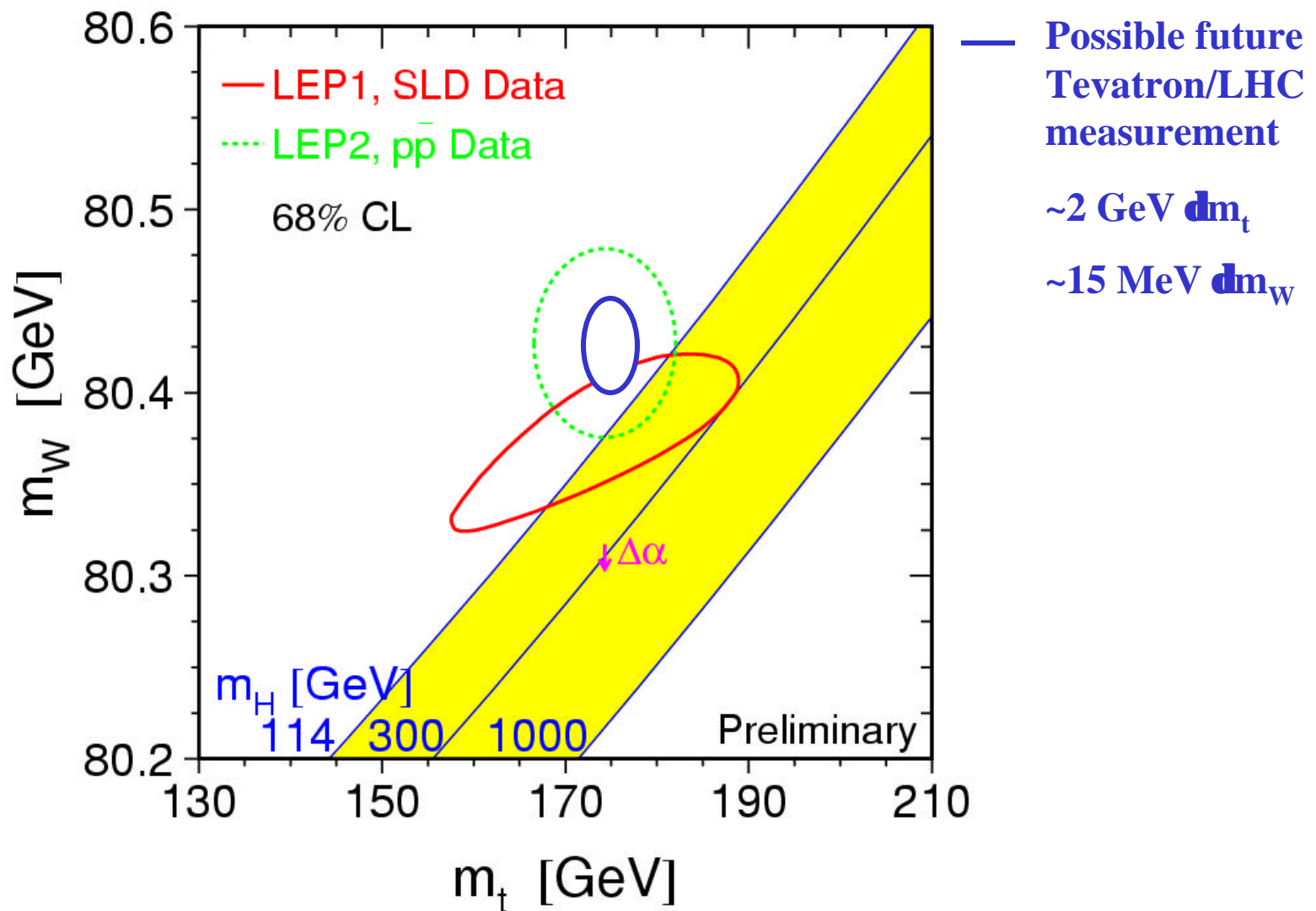
$$m_t = 180.1 \pm 3.6 \pm 4.0 \text{ GeV}$$

P. Renton hep-ph/0206231

fit	m_H GeV	χ^2/df
now: $\delta m_t = \pm 5.1$ GeV, $\delta m_W = \pm 33$ MeV	85^{+54}_{-34}	29 / 15
if $\delta m_t = \pm 2.0$ GeV, $\delta m_W = \pm 33$ MeV	83^{+38}_{-28}	29 / 15
if $\delta m_t = \pm 5.1$ GeV, $\delta m_W = \pm 15$ MeV	67^{+40}_{-27}	34 / 15
if $\delta m_t = \pm 2.0$ GeV, $\delta m_W = \pm 15$ MeV	50^{+21}_{-16}	35 / 15
if $\delta m_t = \pm 1.0$ GeV, $\delta m_W = \pm 10$ MeV	35^{+12}_{-10}	38 / 15

The Tevatron
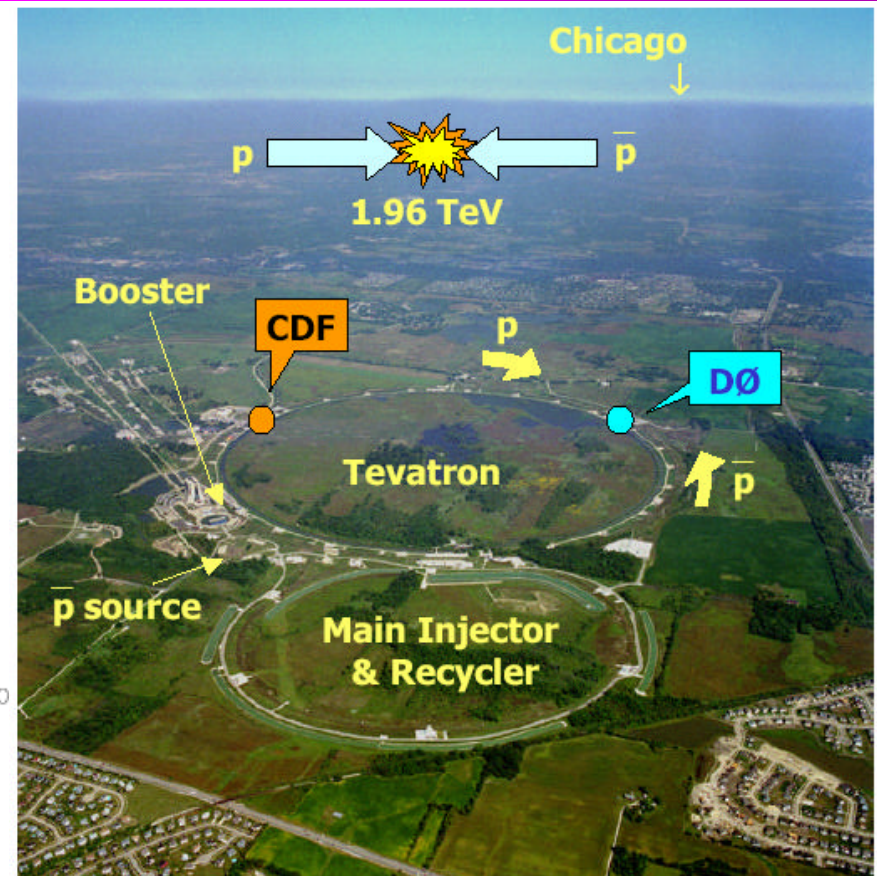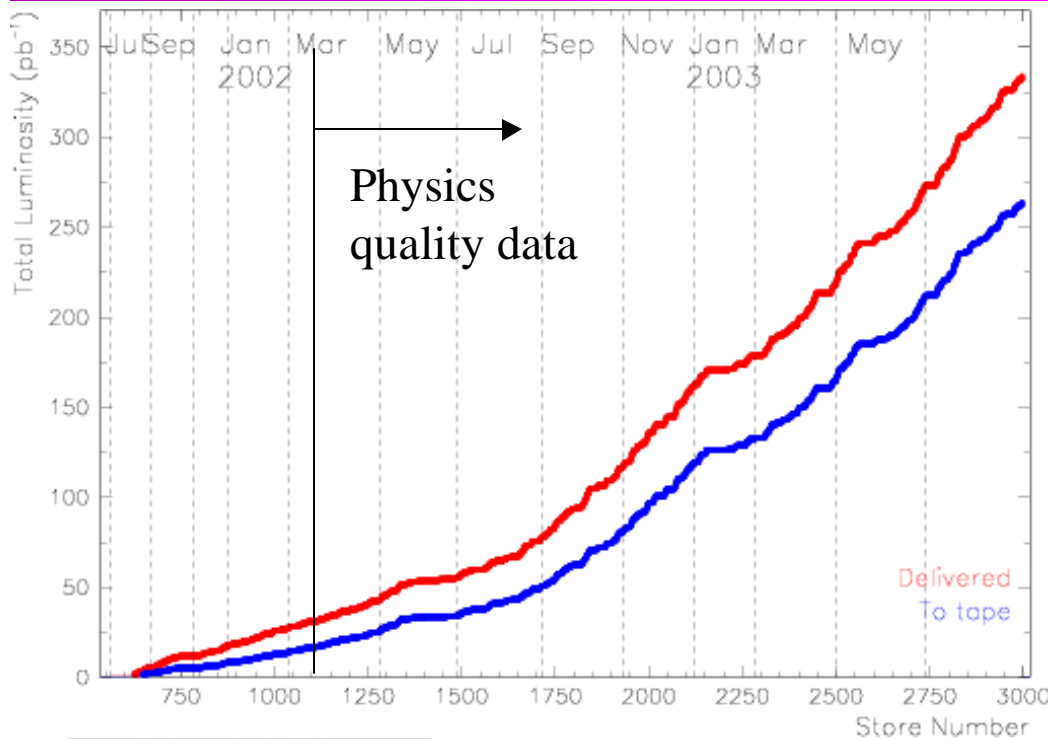
Integrated Luminosity (fb⁻¹)				
	Design Projection		**Base Projection**	
	per year	Accum-ulated	per year	Accum-ulated
FY03	0.22	0.30	0.20	0.28
FY04	0.38	0.68	0.31	0.59
FY05	0.67	1.36	0.39	0.98
FY06	0.89	2.24	0.50	1.48
FY07	1.53	3.78	0.63	2.11
FY08	2.37	6.15	1.14	3.25
FY09	2.42	8.57	1.16	4.41

~330 pb⁻¹ delivered to date

~260 pb⁻¹ on tape

~180 pb⁻¹ for l+jets mass w/ silicon now

~500 pb⁻¹ for Spring 2005 thesis?
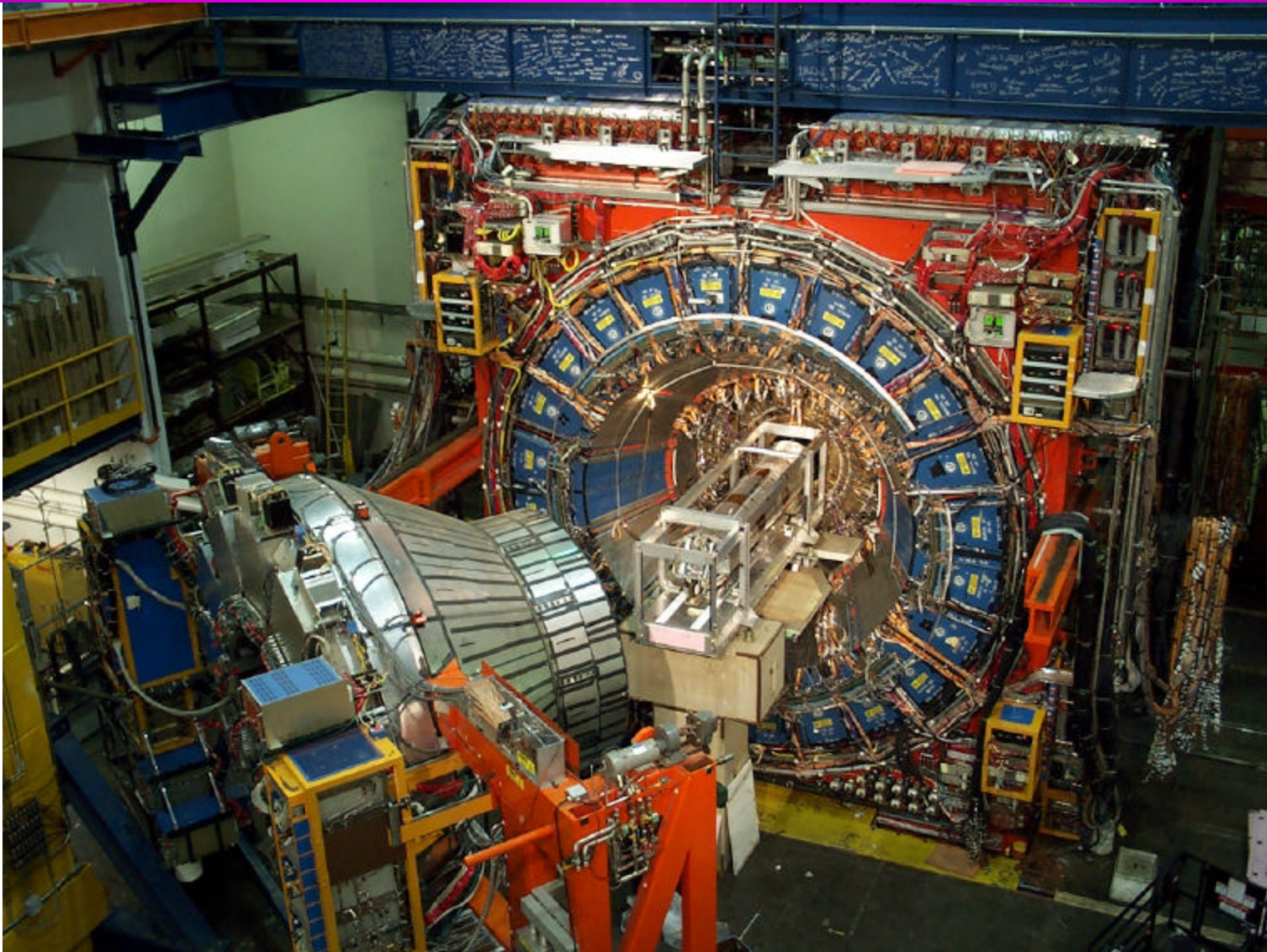
t-t̄ Overview

- Production in p-pbar collisions at $\sqrt{s} = 1.96$ TeV, NLO $\sigma \approx 6.7$ pb

  Cacciari et al hep-ph/0303085

- 85% q-qbar, 15% gluon fusion

- $\Gamma_t \approx 1.4$ GeV, $\tau \approx 10^{-24}$ s

- Leptons (e, μ) well measured

- Quarks (jets) poorly measured

  – And much QCD background

  – B quarks (mesons) taggable

- Neutrinos don't interact in detector

  – Measured indirectly

t-tbar Topologies

Dilepton (e, μ)	l+jets (e, μ)	All Hadronic	Other (τ's)
5%	30%	45%	20%
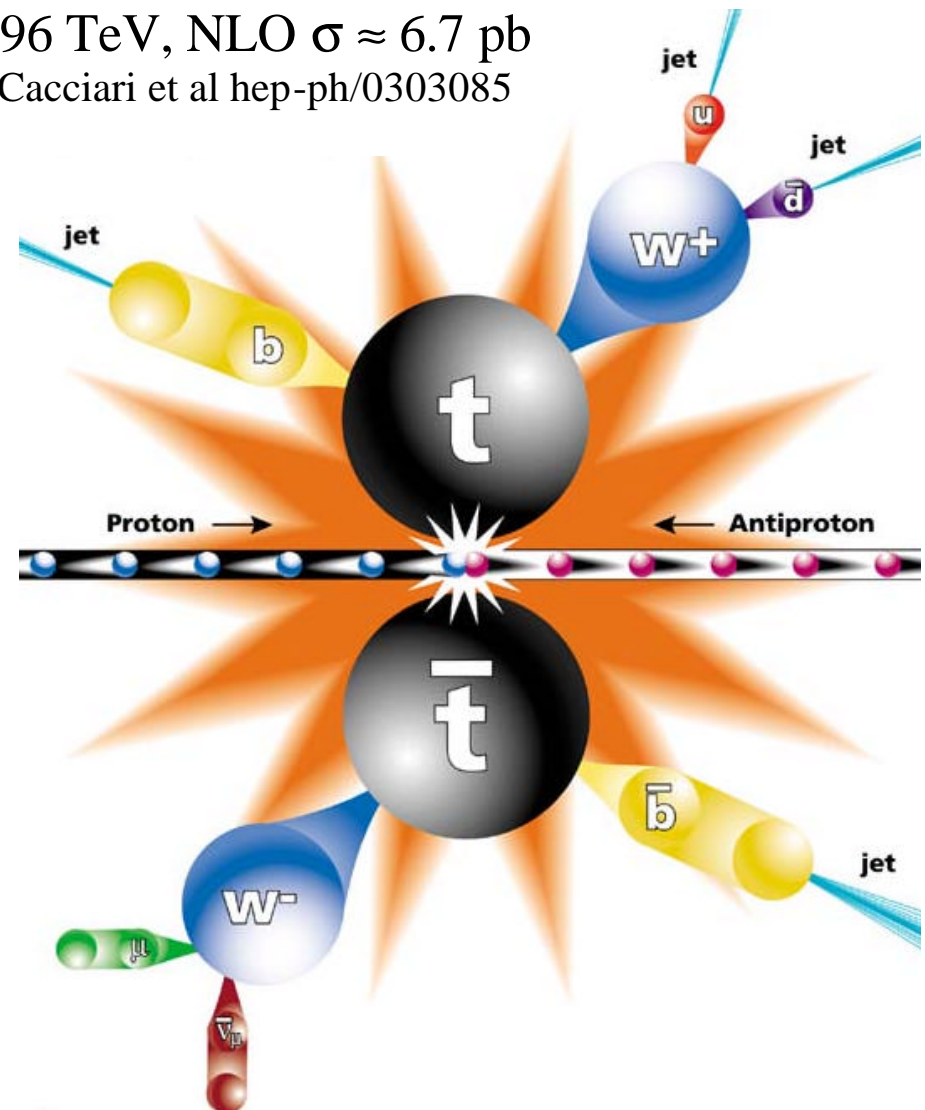
- High E_T (20 GeV) e or μ
- High $\not{E}_T$ (20 GeV)
  - infer ν
- 4 High E_T jets
  - At least one w/ displaced vertex B tag
- Combinatorics – which jets are from t?
- Combinatorics and jet energy measurements make m_t a difficult measurement

CDF II Preliminary

$m_{top}^{event} = 178.9$ GeV/c^2
$m_W^{event} = 94.9$ GeV/c^2

Jet1 102 GeV

Muon 50.5 GeV

$L_{xy} = 6.2$ mm
10.8 s

I.P

MET 40.2 GeV

Jet4 36.5 GeV

$L_{xy} = 4.1$ mm
21.9 s

Jet3 57.8 GeV

Jet2 69.7 GeV

Y (cm)

X (cm)
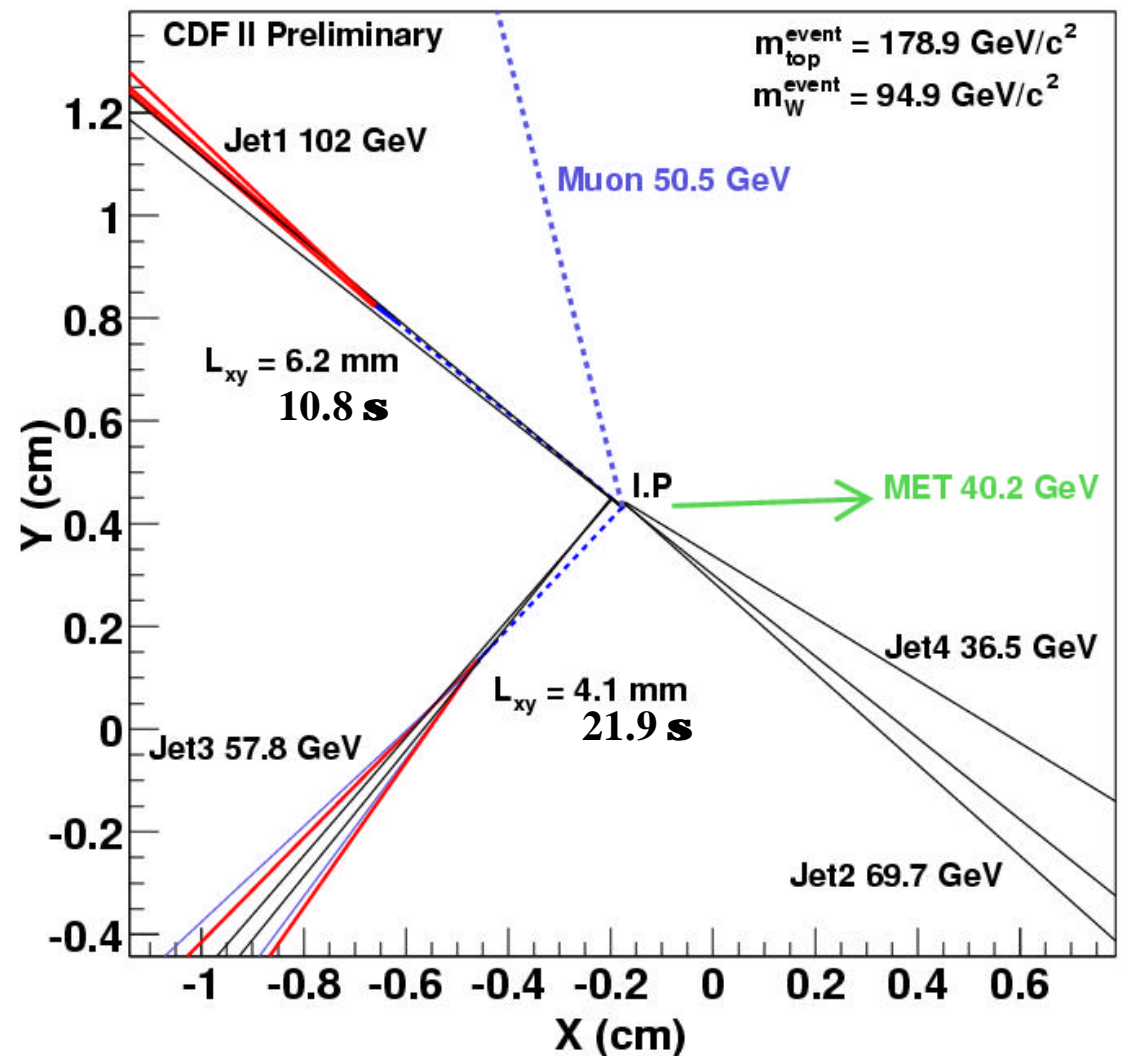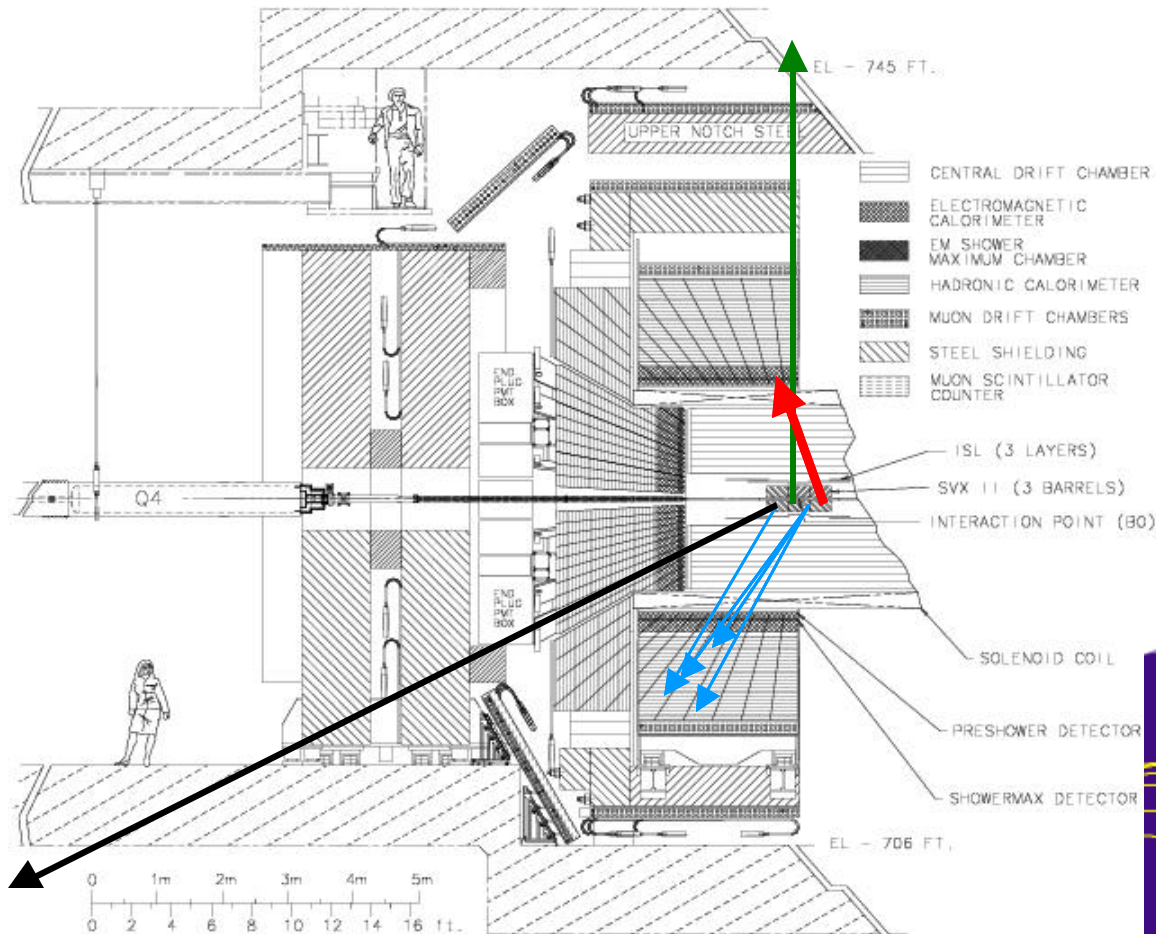
CDF Detector

e's – EM calorimeter

– tracking chamber

μ's – tracking chamber

– muon chambers

Jets – calorimeters

B Jets – calorimeters

– silicon detectors

ν's – don't interact in detector
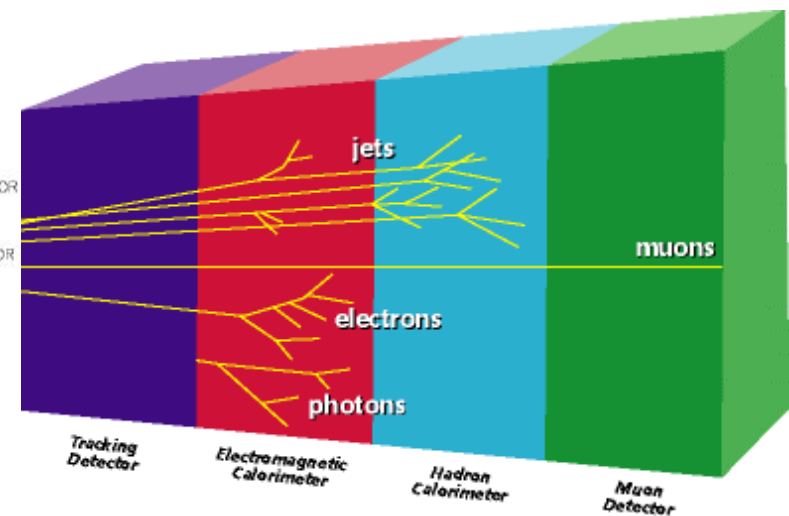
Event by event reconstruction

- 4 jets, 1 central e or μ, large missing E_T, at least one displaced vertex b tag

- 2x3(x2) ways to assign jets to partons with one tag, 2(x2) for double

- Enough measurements to overconstrain system

- 2-C fit to find the (one) best combination (lowest χ^2)
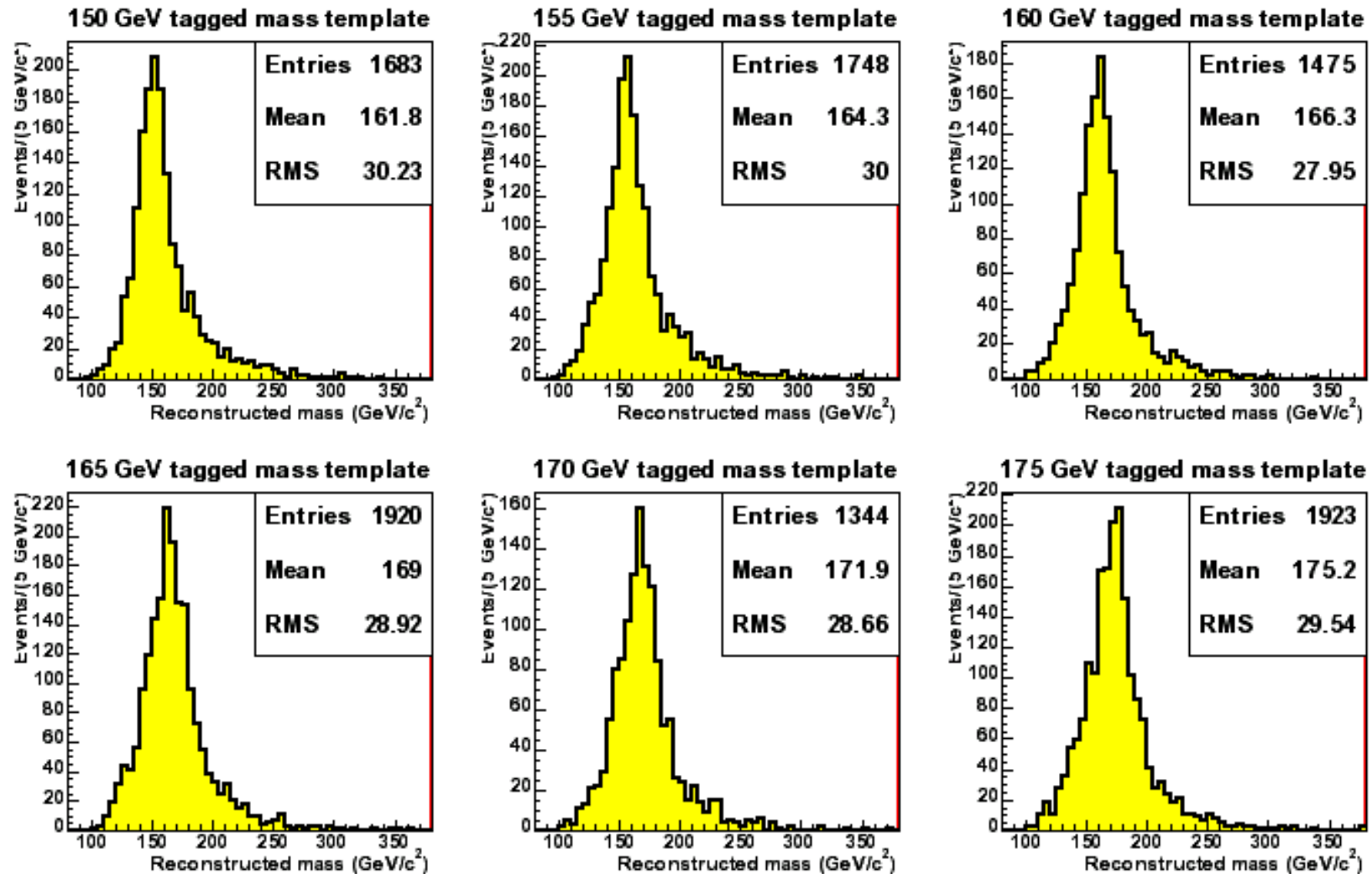
- χ^2 cut to help reject backgrounds

175 GeV tagged mass template

Entries	1923
Mean	175.2
RMS	29.54
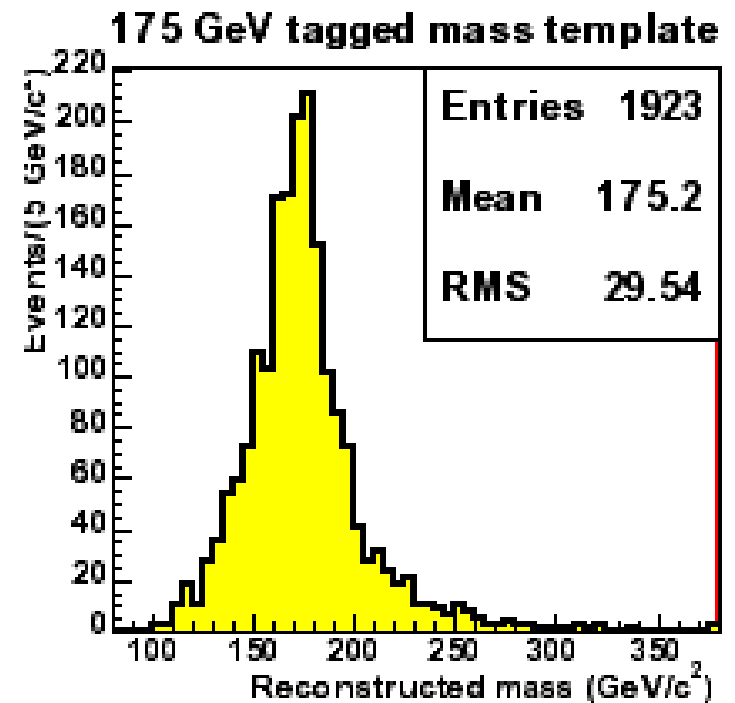
$$c^2 = \sum_{i=l,\,jets} \frac{(p_T^{i,fit} - p_T^{i,meas})^2}{s_i^2} + \sum_{j=x,y} \frac{(p_j^{UE,fit} - p_j^{UE,meas})^2}{s_j^2}$$

$$+ \frac{(M_{jj} - M_W)^2}{\Gamma_W^2} + \frac{(M_{l\nu} - M_W)^2}{\Gamma_W^2} + \frac{(M_{bjj} - M_t)^2}{\Gamma_t^2} + \frac{(M_{bl\nu} - M_t)^2}{\Gamma_t^2}$$

# Build Mass Templates for Various Masses

CDF Run II Preliminary

Source	Events	Approximate σ
W + jets (mistags)	2.25 ± 0.32	33 pb
Wbbar	1.71 ± 0.51	0.74 pb
Wccbar	0.72 ± 0.25	1.39 pb
Wc	0.63 ± 0.13	1.96 pb
WW/WZ	0.20 ± 0.06	
Non-W (QCD)	2.4 ± 0.36	Very Large
Single top	0.4 ± 0.04	
Total	8.31 ± 0.76	

Compare to ttbar cross section of ~7 pb

Background mass template

CDF Run II Preliminary
- W + mistags, non-W
- W + heavy flavor
- Single top, s channel
- Single top, t channel

Fraction/(5 GeV/c^2) vs Reconstructed mass (GeV/c^2)

- b tagging involves a choice between efficiency and fake rate
  - Choice determines background composition
- Overall S:B is 2.7:1
  - 16:6
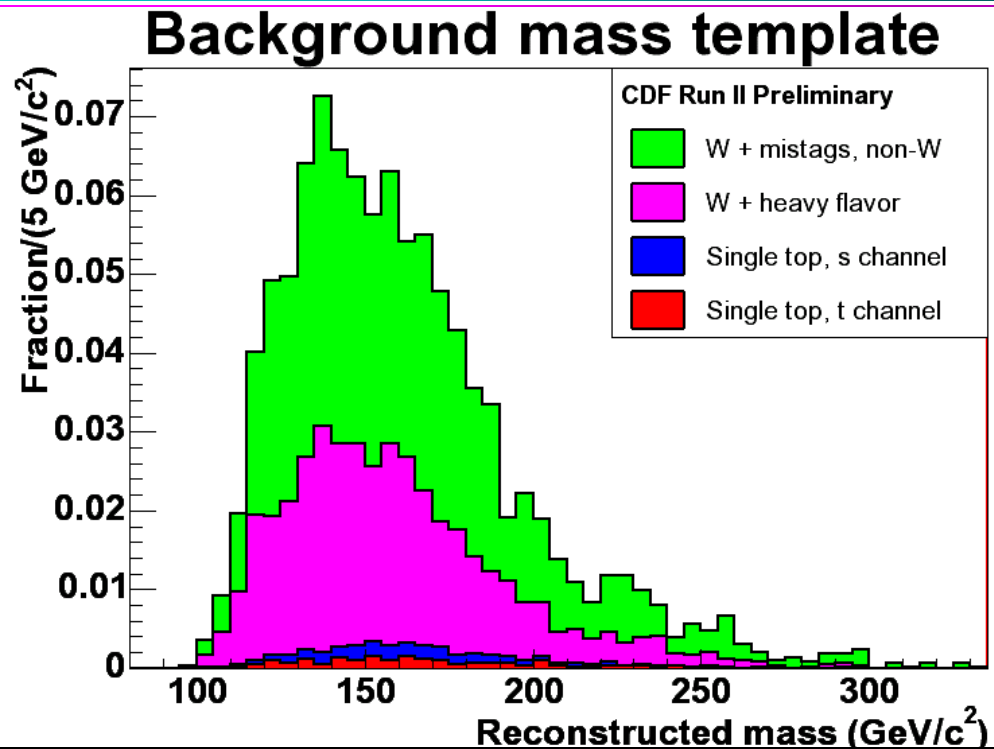
| Background events with $\chi^2 < 10$		
Mass Template Source	Background Source	Number of Events
W + jets (mistags)	Mistags, QCD	3.42 ± 0.36
Wbbar	Wbbar, Wccbar, Wc, WW/WZ	2.26 ± 0.41
Single top: s channel		0.16 ± 0.01
Single top: t channel		0.11 ± 0.01
Total		5.94 ± 0.55

- Signal shape parameterized, and as function of top mass.

- Background shape parameterized

- Unbinned likelihood fit to parameterized templates, with a background constraint
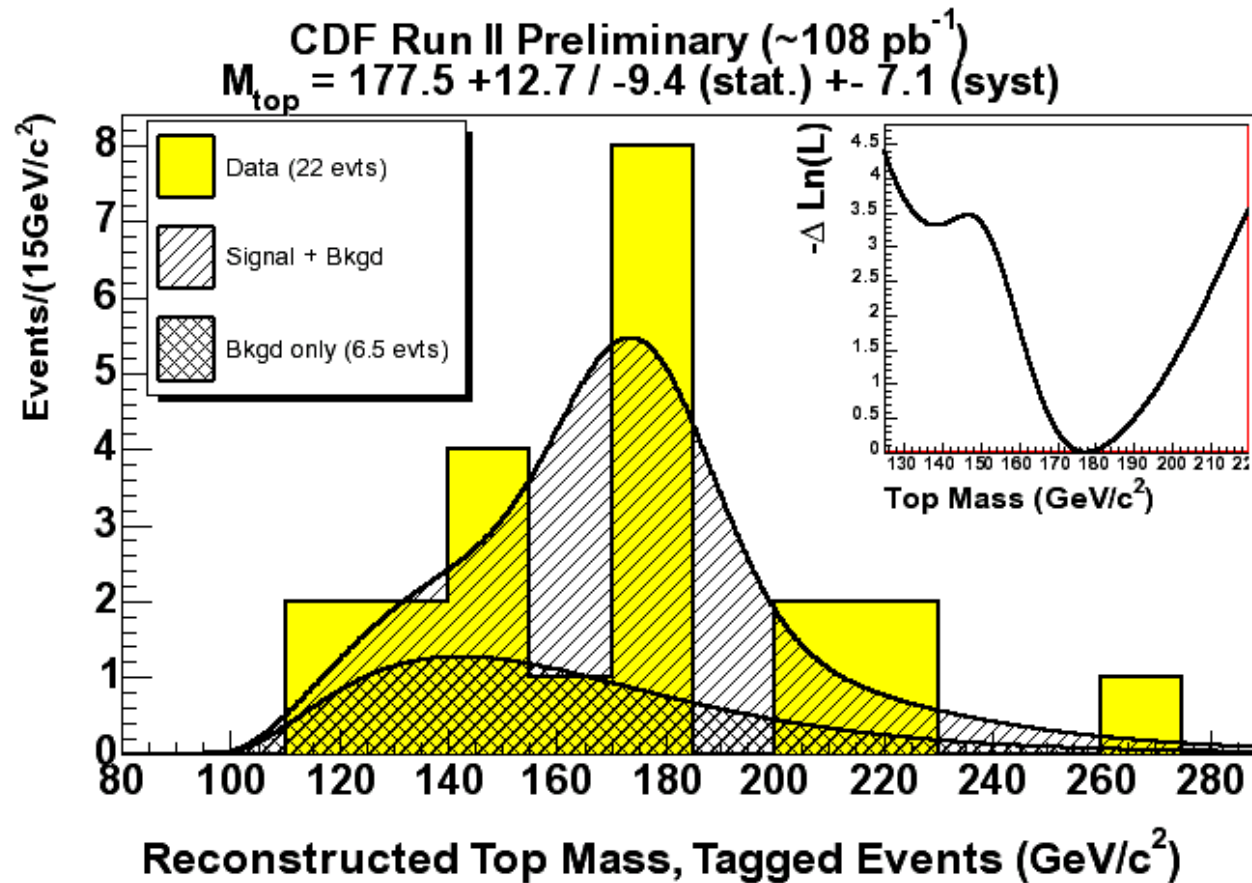
Systematics

- Jet energy measurement leads to dominant systematic
- Initial State and Final State Radiation (ISR/FSR) since gluons affect jet energy, top and W mass, etc.
  - Run I numbers (turn on, off) for now.
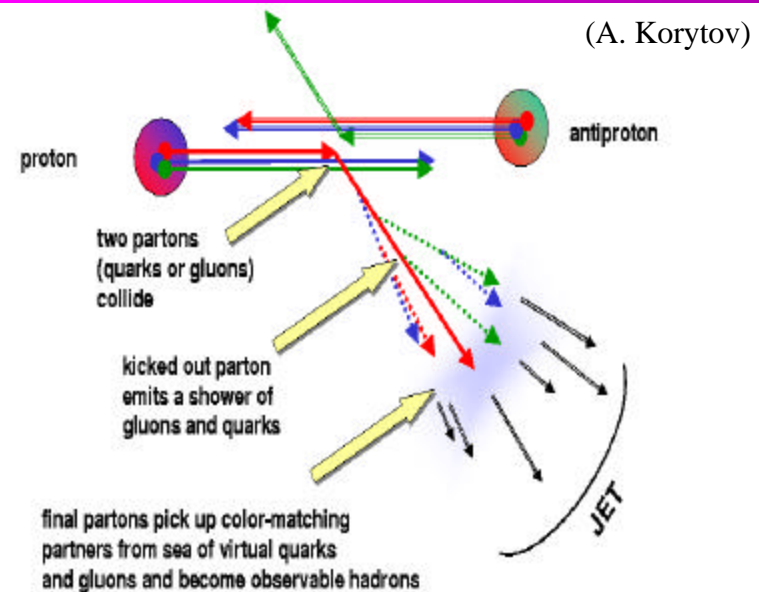- PDF's use CTEQ6M eigenvector sets

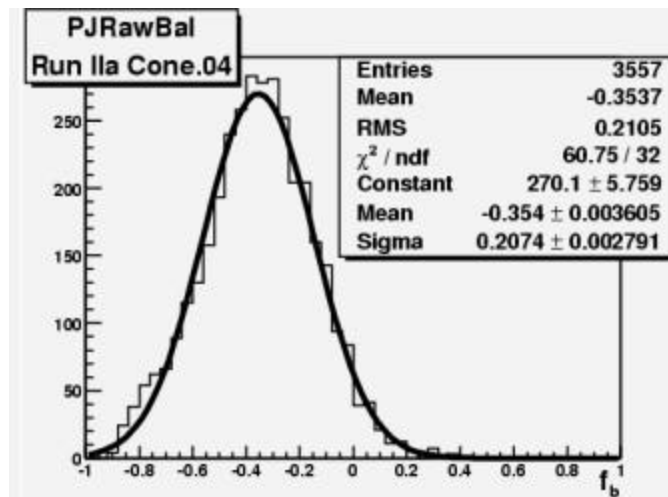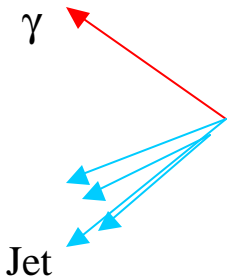| Source of Syst.	ΔM_{top} (GeV/c^2)
Jet Energy	6.2
ISR	1.3
FSR	2.2
Generators	0.5
PDFs	2
Other MC modeling (Jet Resolution, p_T^{top})	1
Background Shape	0.5
b-tagging	0.1 (Run I)
Total	7.1

Understanding Jets

- Quarks and gluons appear in the calorimeter as jets: collection of hadrons
  - Mostly π's

- Reconstruct m_t in terms of parton energies, want to correct jets back to parton level

- Difficult to calibrate at low particle energies typical in jets
  - Cracks in detector, Non-linearities
  - Understanding fragmentation
  - Out of cone energy

(A. Korytov)
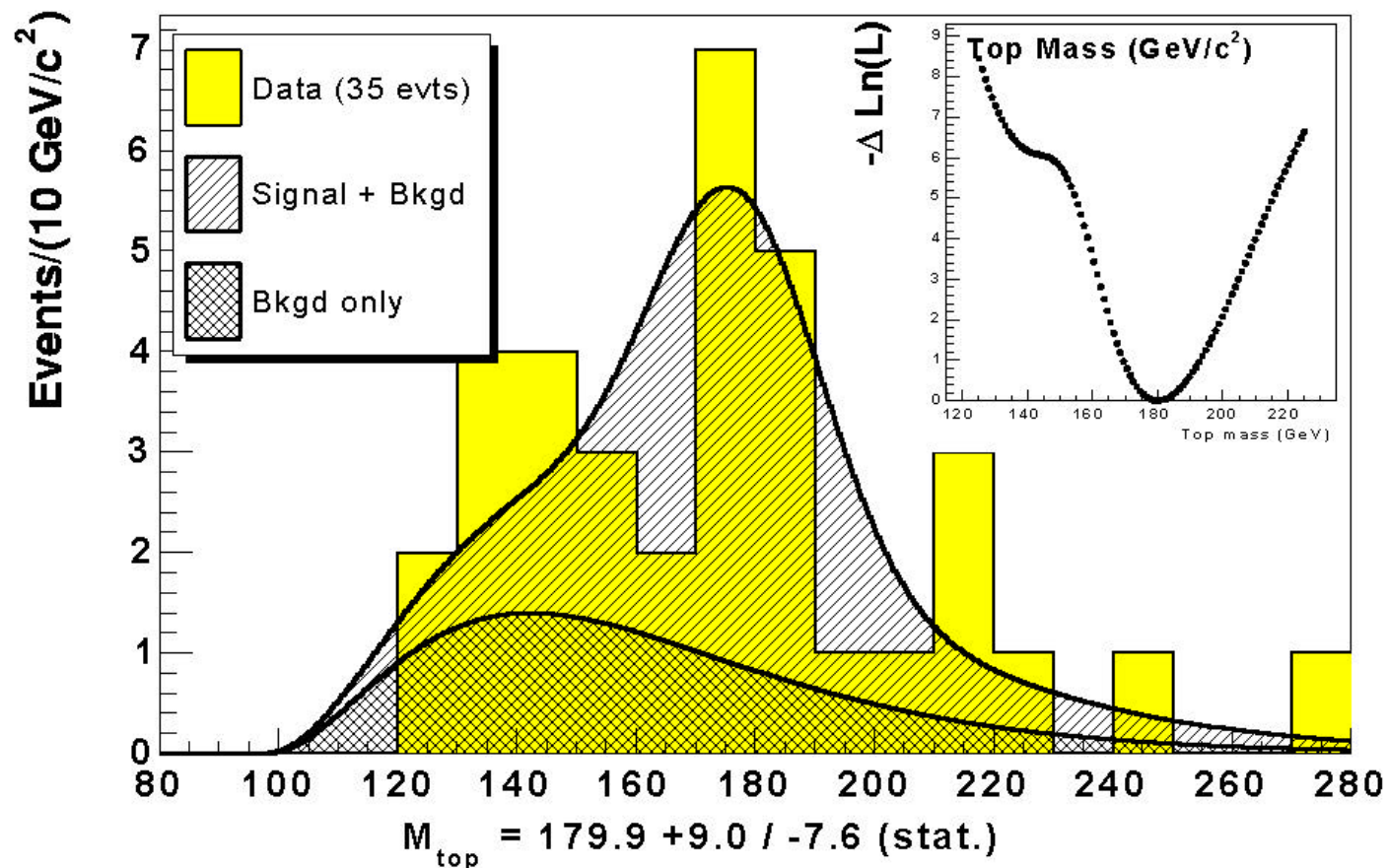

$$f_b = \frac{P_T^{Jet}}{P_T^{\tilde{a}}} - 1$$

γ

Jet

PJRawBal Run IIa Cone.04

Entries	3557
Mean	-0.3537
RMS	0.2105
χ² / ndf	60.75 / 32
Constant	270.1 ± 5.759
Mean	-0.354 ± 0.003605
Sigma	0.2074 ± 0.002791

$<f_b^{raw}>$ vrs. η_J^D Cone 0.4

$$\varsigma = -\ln[\tan(\frac{\grave{e}}{2})]$$
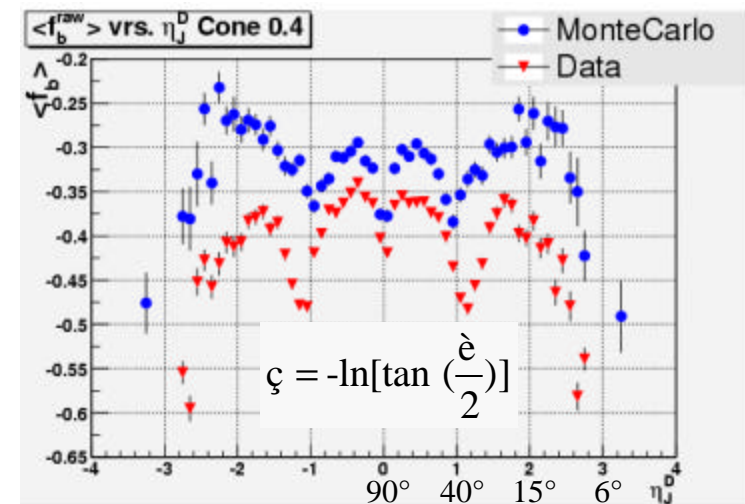
$\sim 180 \text{ pb}^{-1}$

Reconstructed Top Mass (GeV/c²)

M_{top} = 179.9 +9.0 / -7.6 (stat.)

New D0 Run I Top Mass Analysis

- Use all of the information you measure well, integrate over things you don't measure well.

- Compare to our best knowledge of the physics – compare to SM differential cross sections.

- Integrate cross sections over quark energies, using MC-extracted transfer functions to connect to measured jet energies.

$$P_{t\bar{t}}(x, m_t) = \frac{1}{\boldsymbol{s}_{tot}} \int d\boldsymbol{s}(y, m_t) dq_1 dq_2 f(q_1) f(q_2) W(x, y)$$

$|\mathbf{M}|^2$

x measured quantities (e.g. jets)

y matrix element quantities (e.g. partons)

f(q) parton distribution functions

W(x,y) transfer functions q_1, q_2 incoming quark energies

D0 l+jets (1998) $m_t = 173.3 \pm 5.6$ (stat) ± 5.5 (syst) GeV/c^2

D0 l+jets (2003) $m_t = 180.1 \pm 3.6$ (stat) ± 4.0 (syst) GeV/c^2

Traditional CDF Template Method vs. New D0 Matrix Element Method

Traditional CDF (Template)

- One m_t per event, equal weight.

- Single best-fit (χ^2) combination.

- Series of eight levels of jet corrections, get mean correct and assume Gaussian shape.

- Global m_t fit from likelihood fit of data to signal and background templates

New D0 (Matrix Element)

- $P(x : m_t)$ for each event, based upon comparison of fifteen kinematic variables (x) to SM matrix elements

- All combinations weighted according to signal probability, and events combined according to signal probability

- Transfer functions connecting parton energies to jet energies in detail

- Global m_t fit from joint likelihood of signal (mass-dependent) and background (mass-independent) probabilities.

Signal Events

Background Event

P$_{signal}$
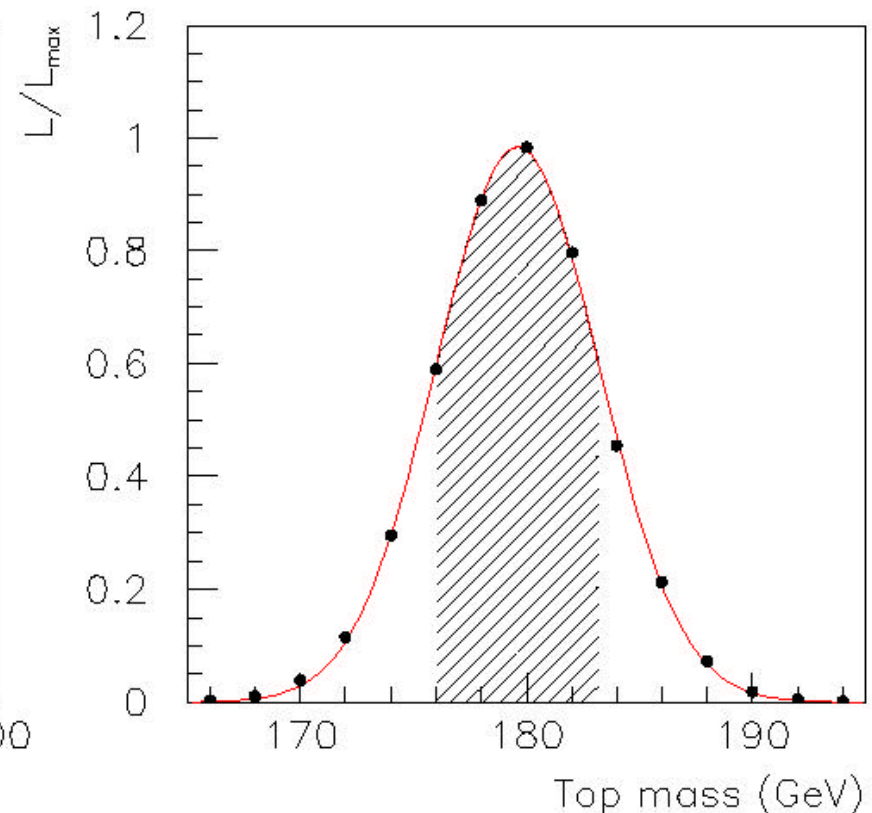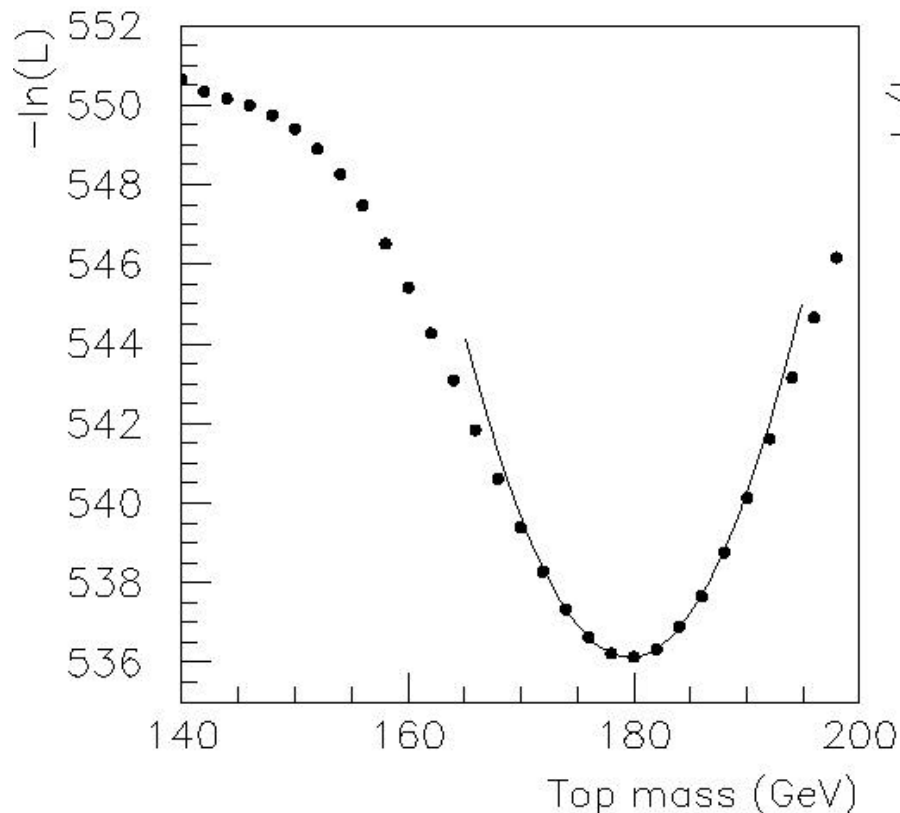
P$_{background}$

M_t = 180.1 ± 3.6 GeV (stat)

Compared with 5.6 GeV statistical error from previous D0 mass analysis.
The statistical error you'd expect from the old D0 analysis with a factor of 2.4 more data.

22 events in data: 12 ± 3 signal (from fit), 10 ± 3 background

Systematics at D0

- D0's new analysis has a significantly smaller systematic due to jet energies.
  - More detailed connection between jets and partons (transfer functions)
- Other systematics smaller as well
  - Using more event information, and combining events and combinations more effectively

D0 (2003)

D0 (1998)

TABLE XXIX. Systematic uncertainty summary.

	LB (GeV/c^2)	NN (GeV/c^2)	Average (GeV/c^2)
Jet energy scale	4.2	3.8	4.0
Generator			
$t\bar{t}$ signal	1.9	1.9	1.9
VECBOS flavors	2.5	2.5	2.5
Noise/MI	1.3	1.3	1.3
Monte Carlo stat.	0.6	1.1	0.85
LB/NN diff	0.8	0.8	0.8
Likelihood fit	1.0	1.0	1.0
Total	5.6	5.4	5.5

Phys. Rev. D {58} 052001 (1998)

Systematic Uncertainties for top quark mass

Determined from MC studies with large event samples:

Signal model	1.5 GeV/c^2
Background model	1.0 GeV/c^2
Noise and multiple interactions PRD 58 52001, (1998)	1.3 GeV/c^2

Determined from data:

Jet Energy Scale	3.3 GeV/c^2
Parton Distribution Function	0.2 GeV/c^2
Acceptance Correction	0.5 GeV/c^2

Total systematic error	4.0 GeV/c^2

Applying D0's Methods at CDF

- Very similar methods have been proposed by CDF members (Kondo) and others (Dalitz and Goldstein)
  - Studied in Run I at CDF, but no mass measurement published.
  - Dynamical Likelihood Method work well underway in Run II

- No magnetic field at D0 Run I
  - Muons poorly measured, integrated over.

- Poor or no silicon coverage at D0 Run I
  - 2003 mass analysis didn't use displaced vertex tags.
  - Easy to use binary SVX tags at CDF, more in keeping with the method to use a tag probability. Either way should help dramatically reduce backgrounds. But, there may be more backgrounds to consider (more matrix elements).

- Straightforward to add extra signal and background matrix elements.

- More difficult to incorporate extra matrix element with gluon radiation, either just extra diagrams or full NLO calculation

D0-Style Transfer Functions at CDF

- We have eight levels of jet corrections at CDF to get from jets back to parton-level quantities.

•In general, their goal is to get the mean right, while assuming a gaussian shape

•Our transfer functions use jets that have been corrected back to particle-jet level (detector effects removed) (level 5 of 8)

•The goal is to start with partons, and accurately model the distribution of jet energies (shape as well as mean)

PartonMinusBJetL5Ecc	
Entries	3714
Mean	11.1
RMS	14.35

$$E_{parton} - E_{jet} \ (GeV)$$

for B jets from ttbar MC

$$n(E_{jet}, E_{parton})dE_{jet}dE_{parton} = n(E_{parton})dE_{parton}W(E_{parton}, E_{jet})dE_{jet}$$

where

$n(E_{parton})$ is the (process dependent) distribution of parton energies

$W(E_{parton}, E_{jet})$ is the probability distribution to have E_{jet} given a E_{parton}

So, we hope to separate the process-dependent $n(E_{parton})$ from the largely process-independent $W(E_{parton}, E_{jet})$

D0 plot from F. Canelli
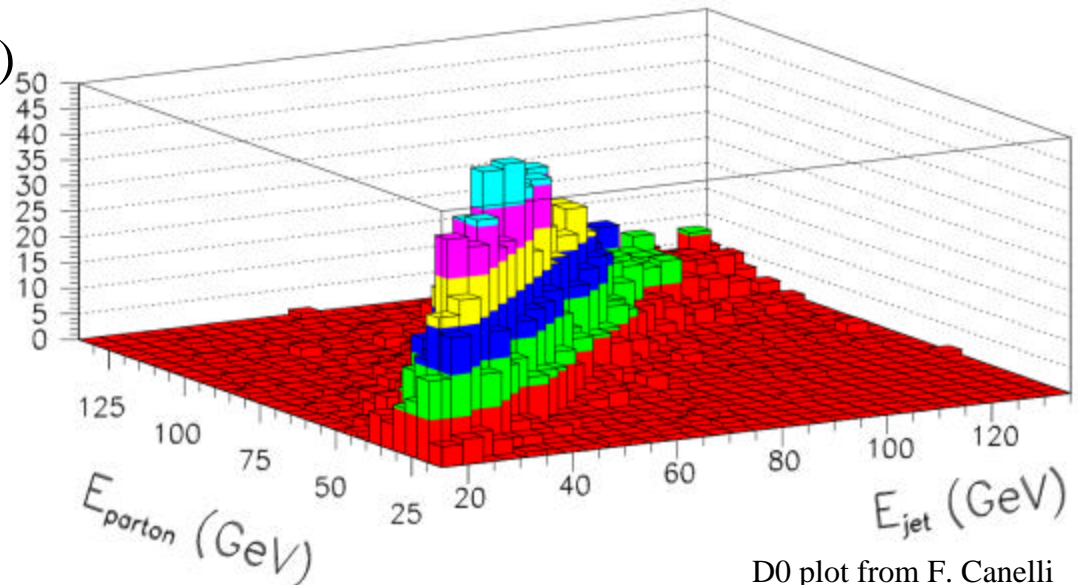
$$d_E = E_{parton} - E_{jet}$$

$$F(d_E) = \frac{1}{\sqrt{2p}\,(p_2 + p_3 p_5)}[\exp\frac{-(d_E - p_1)^2}{2p_2^2} + p_3 \exp\frac{-(d_E - p_4)^2}{2p_5^2}]$$

Testing the transfer functions

Parton information along with the transfer functions (previous page) allow us to make predictions (**blue curves**) of jet level quantities, and compare with simulation (histograms)
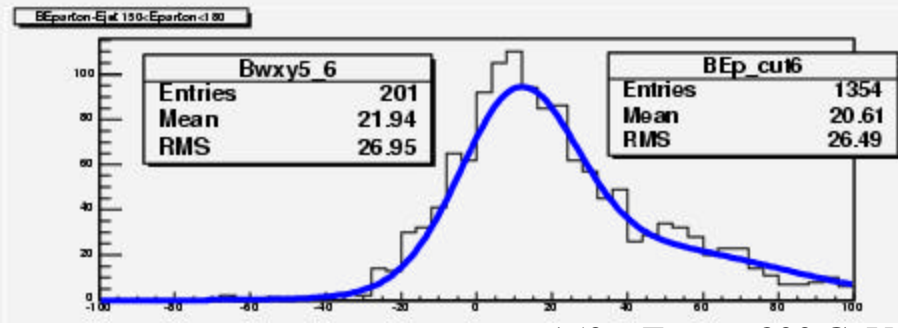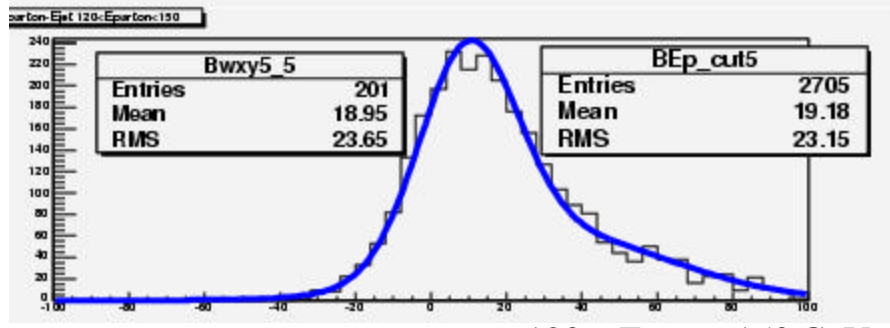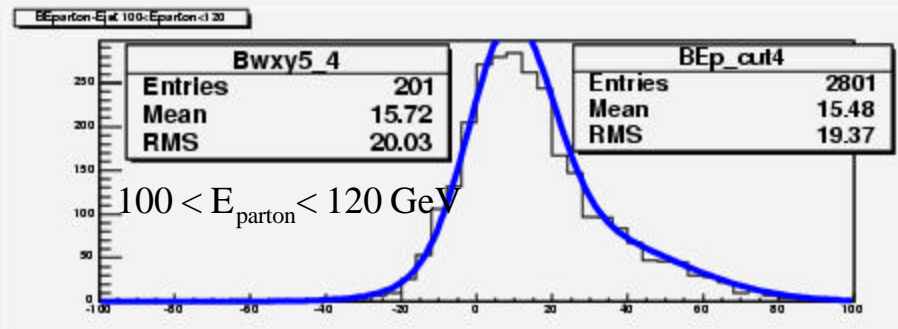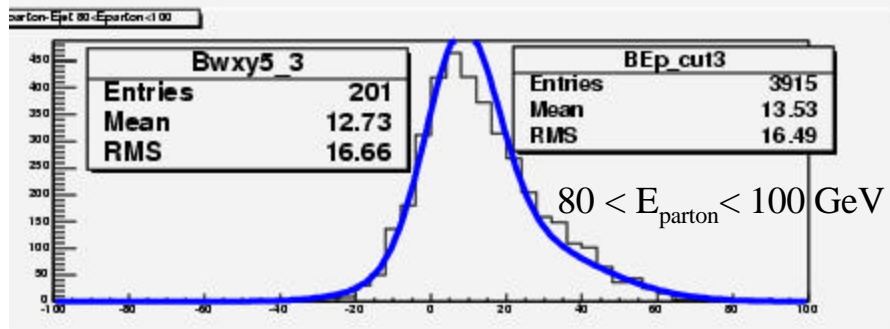

E_{parton} (GeV) from b quarks

prediction	
Entries	60
Mean	94.84
RMS	39.6

PartonFromBJetL5Ecc	
Entries	20538
Mean	96.02
RMS	41.42

E_{jet} (GeV) for b quarks

Bxy2	
Entries	241
Mean	81.56
RMS	38.79

JetFromBJetL5Ecc	
Entries	20538
Mean	82.97
RMS	40.89

$\delta E = E_{parton} - E_{jet}$ for b quarks

Bxy1	
Entries	101
Mean	10.16
RMS	14.15

PartonMinusBJetL5Ecc	
Entries	20538
Mean	10.72
RMS	14.41
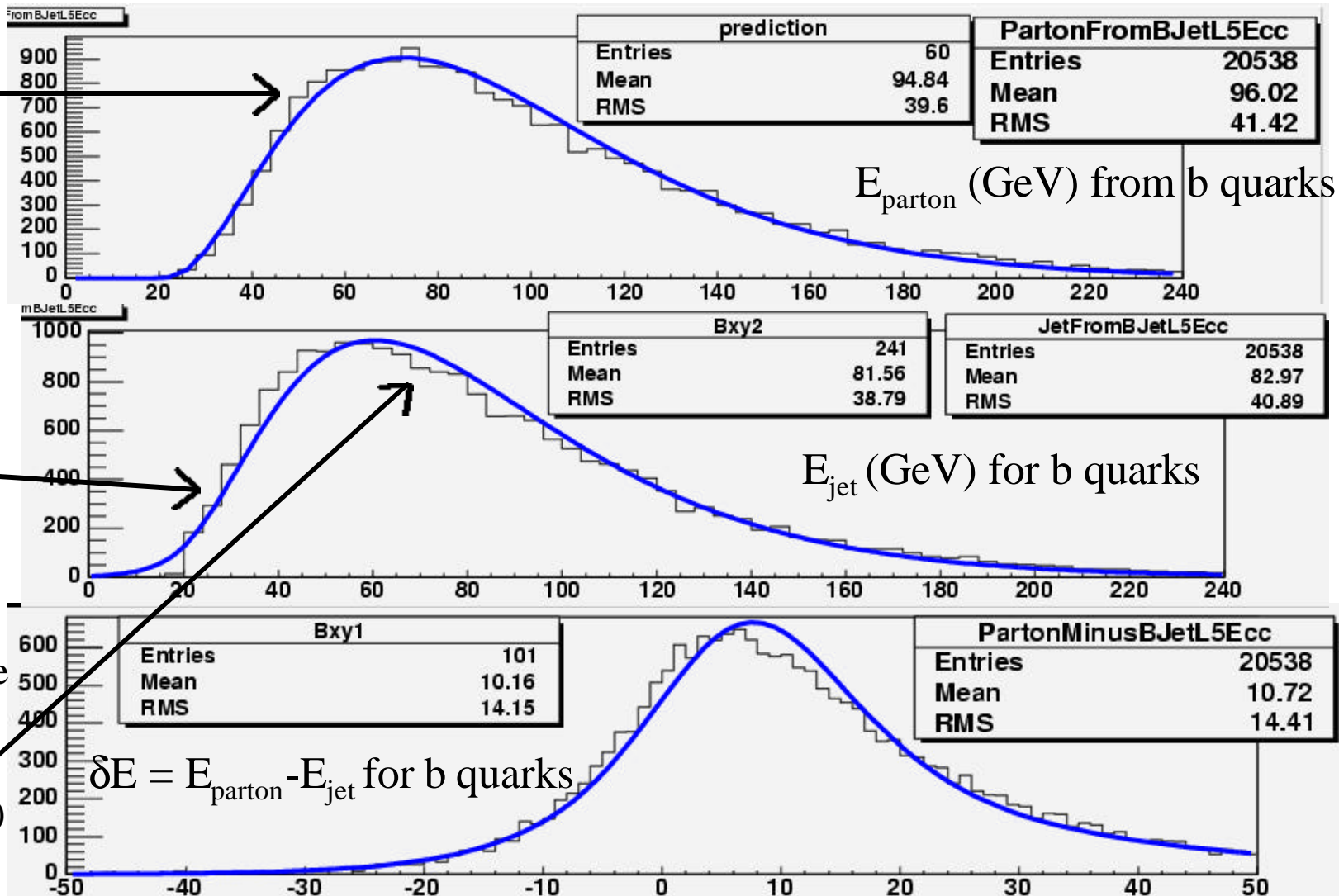
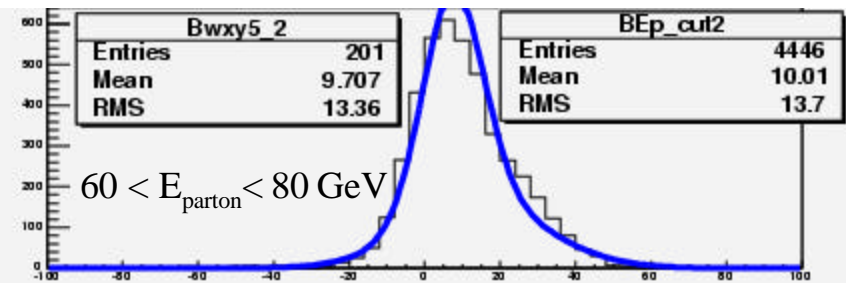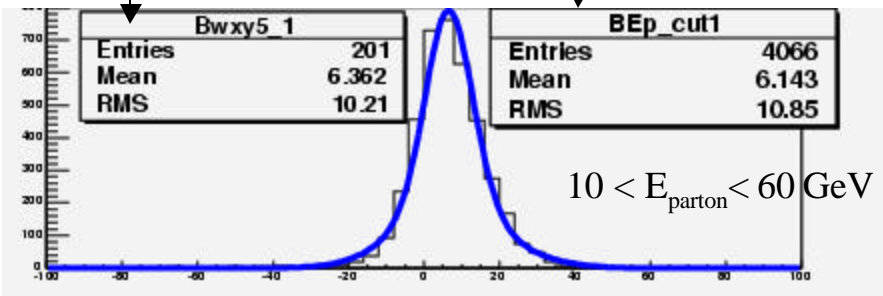# $\delta E = E_{parton} - E_{jet}$ (GeV) for b quarks
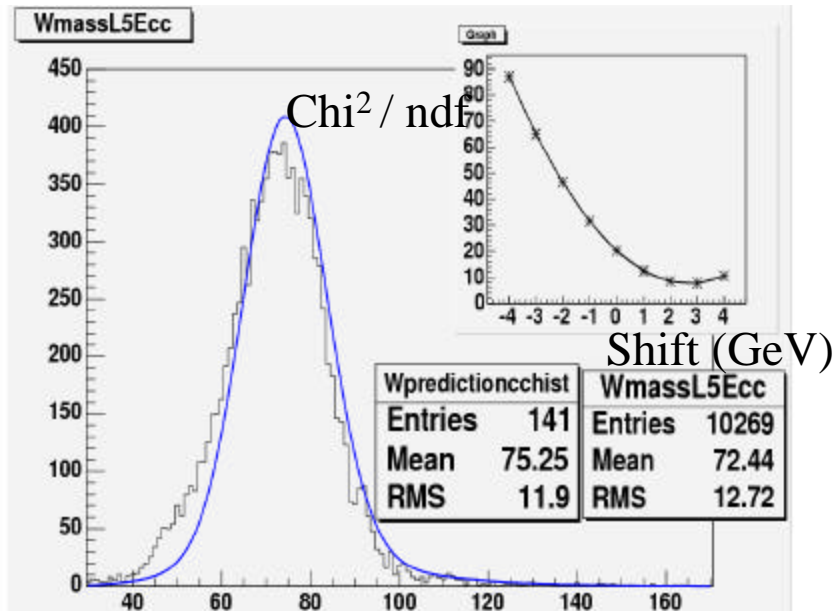
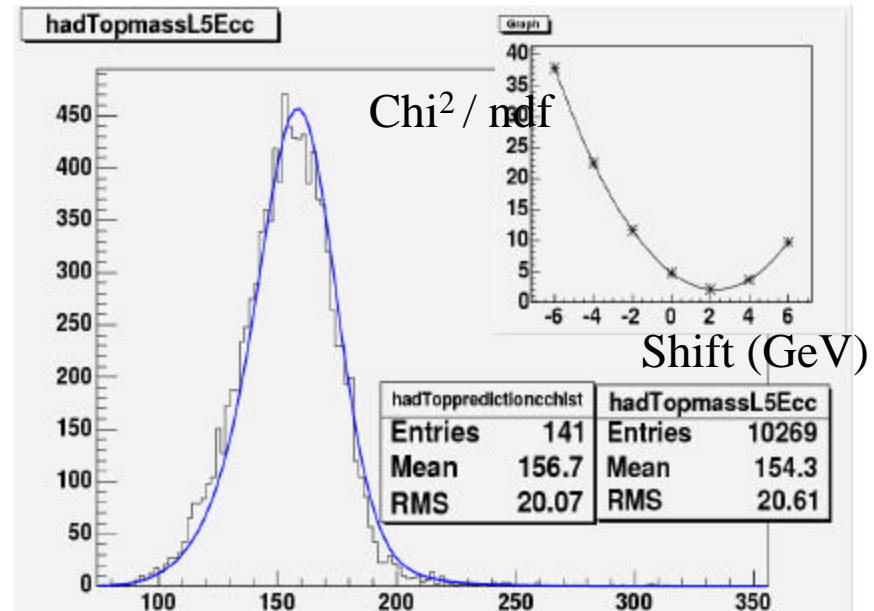prediction   simulation, reconstruction

Hadronic W and top mass from transfer functions – correct combination

Hadronic W mass (GeV)   Hadronic top mass (GeV)

- Histogram from simulated, reconstructed Herwig jets.

- **Blue curve** is prediction from transfer function, using parton level Herwig.

- Inset is chi^2 as we shift the histogram against the prediction.

- Prediction is systematically high.

- Examples of the transfer functions D0 used for their analysis
- D0 saw a small bias also and was able to show that it didn't significantly affect the final top mass measurement (took a 0.5 GeV shift)
- Showed that ttbar transfer functions worked with background samples

ttbar MC events, hadronic top mass   W+jets MC events, 3 jet invariant mass
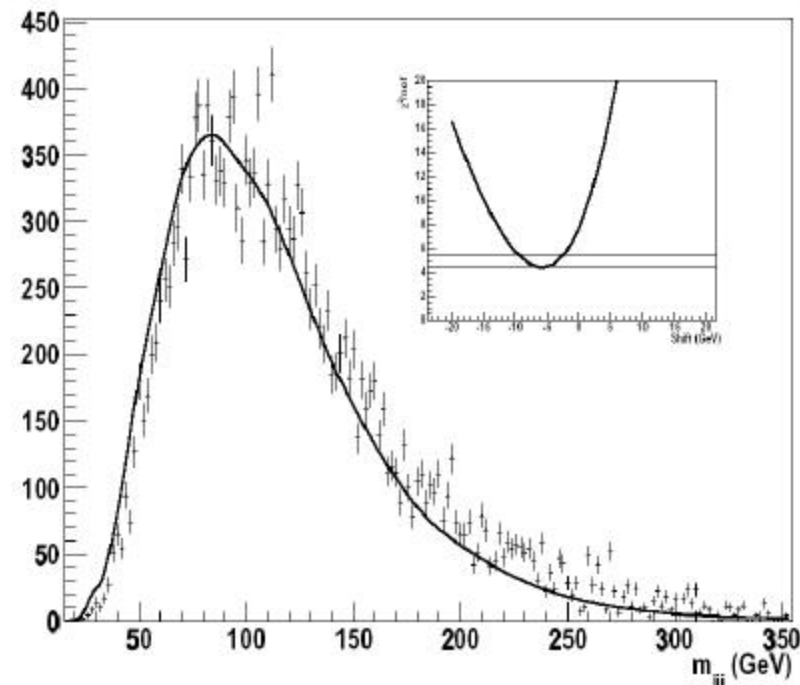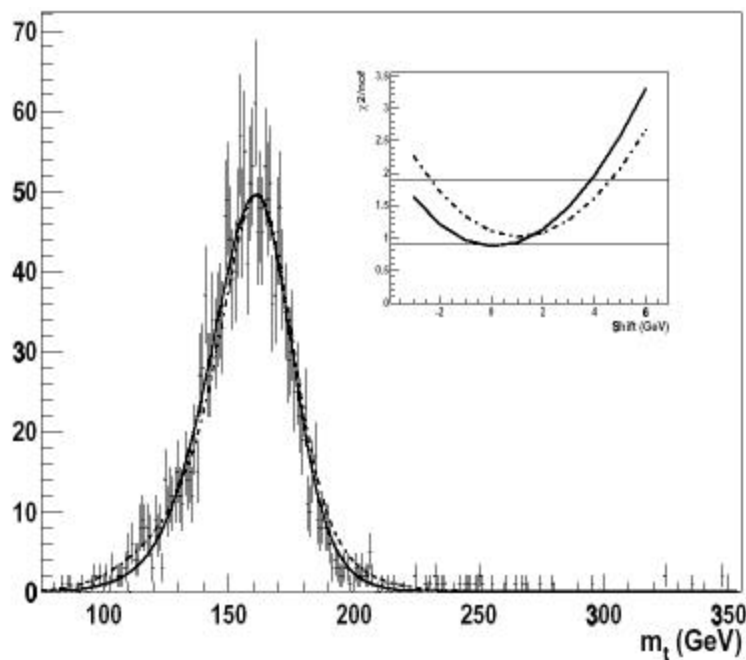
- 1-D Template Method, Run I CDF Method
  - Stat error 4.1 – 5.0 GeV (scale mean expected Run II error, scale current Run II error)
  - Syst error: with no brand new methods (W->qq, Z->bb calib) perhaps 5.0 GeV, with new methods, and reinterpretation of ISR/FSR, perhaps 3.0 GeV
  - Total error 5.1 – 7.1 GeV

- Matrix element method
  - Stat error 2.6 – 3.2 GeV (scale CDF error by factor of $\sqrt{2.4}$ more stat power)
  - Syst error – scale template method systematics by 0.73?
    2.2 – 3.7 GeV (or perhaps as large as template method, 5.0 GeV)
  - Total error 3.4 – 4.9 GeV (or 5.9 GeV)

- The lower statistical error is of short term interest (while statistics are still very limited)
  - Always nice to make your statistical error as small as possible

- Possibility for smaller systematics intriguing for the medium and long-term.

Summary

- The top mass is interesting in and of itself
- Especially interesting as a precision EW observable
  - Constrain SM, predict SM Higgs mass
  - Constrain physics beyond the SM
- I've participated in a template based mass analysis
  - $m_t = 177.5^{+12.7}_{-9.8}$ (stat) ± 7.1 (syst) GeV/c^2 (108 pb^{-1})
- Will continue to contribute to important tools like γ-jet balancing.
- Will pursue a matrix-element based analysis with the prospect of substantially improving the statistical power of the data we collect while also lowering the systematic uncertainty.
  - $m_t = 1xx.x \pm 2.6$ (stat) ± 3.7 (syst) GeV/c^2 ?? (500 pb^{-1})